Foundation for the Atlantic Canada Mathematics Curriculum

Mathematics

FOUNDATION

ISBN 1-895669-27-8

PO Box 2044
Halifax NS B3J 2Z1
Ph: (902) 424-5352
Fax: (902) 424-8976
E-mail: premiers@fox.nstn.ca

Acknowledgments

The Departments of Education of New Brunswick, Newfoundland and Labrador, Nova Scotia and Prince Edward Island gratefully acknowledge the contributions of the following groups and individuals toward the development of this document:

- The regional mathematics common curriculum committee, which has overseen the common curriculum development and provided direction with respect to the completion of this foundation document. Current and past members include the following:

PROVINCE OF NEW BRUNSWICK

John Hildebrand, Mathematics Consultant,
Department of Education

Carolyn Jones, Mathematics Teacher,
Fredericton High School

Joan Manuel, Mathematics/Science Supervisor,
School District 10

Jack Powers, Vice-Principal,
MacNaughton Science and Technology Centre

PROVINCE OF NEWFOUNDLAND AND LABRADOR

Patricia Maxwell, Program Development Specialist,
Department of Education

Sadie May, Mathematics Teacher,
Deer Lake-St. Barbe South Integrated School Board

Ed Somerton, Mathematics Consultant,
Department of Education

Donald Squibb, Mathematics Teacher,
St. James Regional High School

PROVINCE OF NOVA SCOTIA

Beth Calabrese, Mathematics Teacher,
Queen Elizabeth High School

Ken MacInnis, Elementary Teacher,
Sir Charles Tupper Elementary School

Richard MacKinnon, Mathematics Consultant,
Department of Education and Culture

Linda Wheadon, Mathematics Teacher,
Horton District High School

PROVINCE OF PRINCE EDWARD ISLAND

Jerry Coady, Mathematics Teacher,
Bluefield High School

Byron Cutcliffe, Senior High Mathematics/Science
Consultant, Department of Education

Wayne Denman, Mathematics Consultant,
Department of Education

Bill MacIntyre, Elementary Mathematics/Science
Consultant, Department of Education

- The mathematics foundation document working group, comprising teachers and other educators in the province of New Brunswick, which served as lead province in drafting and revising the document.
- The provincial working groups, comprising teachers and other educators in New Brunswick, Newfoundland and Labrador, Nova Scotia and Prince Edward Island, who provided feedback and input to the document during the development and revision process.
- The educators, parents and other stakeholders, who contributed many hours to the validation process which led to the finalization of the *Foundation for the Atlantic Canada Mathematics Curriculum.*

Table of Contents

Vision

The Atlantic Canada mathematics curriculum
is shaped by a vision which fosters the development
of mathematically literate students who can extend
and apply their learning and who are effective participants
in an increasingly technological society.

Introduction

PURPOSE OF DOCUMENT

The development of this foundation document has been motivated by the Atlantic Canada common curriculum project in mathematics. Within this context the document is intended to

- provide both educators and members of the general public with an outline of the philosophy, scope and outcomes of mathematics education in Atlantic Canada
- provide a foundation upon which educators and others will make decisions concerning learning experiences
- serve as a guide to inform the subsequent development of detailed curriculum guides at all grade levels from school entry through grade 12
- articulate a progression and continuity in the development of mathematical skills and concepts from school entry through grade 12, and encourage discussion among mathematics educators at various grade levels regarding this development

CURRICULUM FOCUS

The philosophy and outcomes of the Atlantic Canada mathematics curriculum are based firmly on those articulated by the National Council of Teachers of Mathematics (NCTM) in its *Curriculum and Evaluation Standards for School Mathematics* (1989).

The Atlantic Canada mathematics curriculum is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in an increasingly technological society.

The mathematically literate student will

- appreciate the utility and value of mathematics
- demonstrate mathematical power, i.e., display confidence and competence in his/her ability to do mathematics
- be a mathematical problem solver
- communicate mathematically
- reason mathematically

Further, a mathematics curriculum should reflect the following realities about the nature of mathematics itself:

- To learn and understand mathematics is to "do" mathematics, i.e., students must take an active role in their study of mathematics.
- Mathematics is a means by which we interpret the world and must be regularly connected to meaningful applications.
- Mathematics instruction, and mathematics itself, have been greatly affected by changes in technology.

An appreciation of the importance of mathematical literacy and of the nature of mathematics itself needs to permeate the breadth and depth of the mathematics curriculum at all instructional levels.

A COMMON APPROACH

In 1993 work began on the development of common curricula in specific core programs. The Atlantic ministers' primary purposes for collaborating in curriculum development are to

- improve the quality of education for all students through shared expertise and resources
- ensure that the education students receive across the region is equitable
- meet the needs of both students and society

Under the auspices of the Atlantic Provinces Education Foundation, development of Atlantic common core curricula for mathematics, science, English language arts and social studies follows a consistent process. Each project requires consensus by a regional committee at designated decision points; all provinces have equal weight in decision making. Each province has procedures and mechanisms for communicating and consulting with education partners, and it is the responsibility of the provinces to ensure that stakeholders have input into regional curriculum development.

Each foundation document includes statements of essential graduation learnings, general curriculum outcomes for that core program, and corresponding outcomes at the end of key stages (entry-grade 3, grades 4-6, grades 7-9, grades 10-12). Essential graduation learnings and general curriculum outcomes provide a consistent vision for the development of a rigorous and relevant core curriculum. In addition to this foundation document, teachers will receive supporting curriculum guides for the grade levels at which they teach.

General curriculum outcomes
are statements which identify what students are expected to know and be able to do upon completion of study in a curriculum area.

Key-stage curriculum outcomes
are statements which identify what students are expected to know and be able to do by the end of grades 3, 6, 9 and 12, as a result of their cumulative learning experience in a curriculum area.

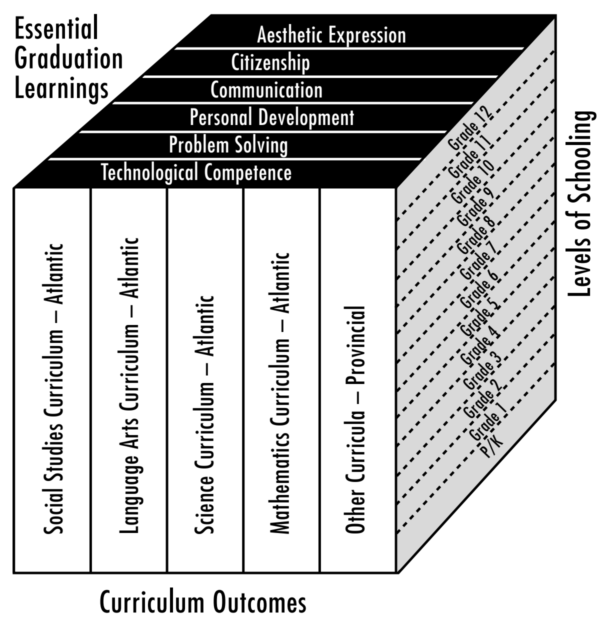
Outcomes

ESSENTIAL GRADUATION LEARNINGS

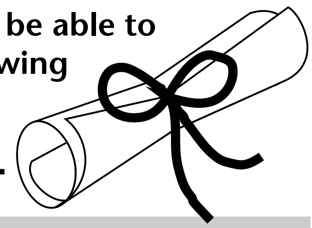
Essential graduation learnings *(EGLs)*
are statements describing the knowledge, skills and attitudes expected of all students who graduate from high school. Achievement of the essential graduation learnings will prepare students to continue to learn throughout their lives. These learnings describe expectations not in terms of individual school subjects but in terms of knowledge, skills and attitudes developed throughout the curriculum. They confirm that students need to make connections and develop abilities across subject boundaries if they are to be ready to meet the shifting and ongoing demands of life, work and study today and in the future. Essential graduation learnings are cross-curricular, and curriculum in all subject areas is focussed to enable students to achieve these learnings. Essential graduation learnings serve as a framework for the curriculum development process.

Curriculum outcome statements
articulate what students are expected to know and be able to do in particular subject areas. These outcome statements also describe the knowledge, skills and attitudes which students are expected to demonstrate at the end of certain key stages in their education as a result of their cumulative learning experiences at each grade level in the entry-graduation continuum. Through the achievement of curriculum outcomes, students demonstrate the essential graduation learnings.

FIGURE 1 – Relationship among Essential Graduation Learnings, Curriculum Outcomes & Levels of Schooling



Graduates from the public schools of Atlantic Canada will be able to demonstrate knowledge, skills and attitudes in the following essential graduation learnings. Provinces may add additional essential graduation learnings as appropriate.



Aesthetic Expression

Graduates will be able to respond with critical awareness to various forms of the arts and be able to express themselves through the arts.

Graduates will be able, for example, to

- use various art forms as a means of formulating and expressing ideas, perceptions and feelings
- demonstrate understanding of the contribution of the arts to daily life, cultural identity and diversity, and the economy
- demonstrate understanding of the ideas, perceptions and feelings of others as expressed in various art forms
- demonstrate understanding of the significance of cultural resources such as theatres, museums and galleries

The mathematics curriculum contributes to the development of aesthetic expression in at least two ways. First, measurement and geometry play a significant role in terms of shape and perspective in the visual arts, architecture and other media. Second, the elegance, power and efficiency of mathematical symbolism and representations are aesthetically satisfying to many.

Citizenship

Graduates will be able to assess social, cultural, economic and environmental interdependence in a local and global context.

Graduates will be able, for example, to

- demonstrate understanding of sustainable development and its implications for the environment
- demonstrate understanding of Canada's political, social and economic systems in a global context
- explain the significance of the global economy on economic renewal and the development of society
- demonstrate understanding of the social, political and economic forces that have shaped the past and present and apply those understandings in planning for the future
- examine human rights issues and recognize forms of discrimination
- determine the principles and actions of just, pluralistic and democratic societies
- demonstrate understanding of their own and others' cultural heritage, cultural identity and the contribution of multiculturalism to society

The citizenship EGL is a major focus of the mathematics curriculum in terms of applications of mathematics. Mathematical applications are of considerable significance in relation to key understandings with respect to government, society and environment. Examples of these applications would range from measurement and geometry in geography to exponential relations in population dynamics and economics and statistical issues in election polling.

Communication

Graduates will be able to use the listening, viewing, speaking, reading and writing modes of language(s) and mathematical and scientific concepts and symbols to think, learn and communicate effectively.

Graduates will be able, for example, to

- explore, reflect on and express their own ideas, learnings, perceptions and feelings
- demonstrate understanding of facts and relationships presented through words, numbers, symbols, graphs and charts
- present information and instructions clearly, logically, concisely and accurately for a variety of audiences
- demonstrate a knowledge of the second official language
- access, process, evaluate and share information
- interpret, evaluate and express data in everyday language
- critically reflect on and interpret ideas presented through a variety of media

Communication is one of the unifying ideas within the mathematics curriculum. Not only does mathematics represent a major means of communication in the sciences and related disciplines, but the mathematics curriculum provides students with a wide variety of communication tools, including diagrams, graphs and tables, as well as a broad range of mathematical symbolism. Through the study of mathematics students will reflect upon and clarify ideas and relationships; express mathematical ideas orally and in writing; and use the skills of reading, listening and viewing to interpret and evaluate mathematical ideas.

Personal Development

Graduates will be able to continue to learn and to pursue an active, healthy lifestyle.

Graduates will be able, for example, to

- demonstrate preparedness for the transition to work and further learning
- make appropriate decisions and take responsibility for those decisions
- work and study purposefully both independently and in groups
- demonstrate understanding of the relationship between health and lifestyle
- discriminate among a wide variety of career opportunities
- demonstrate coping, management and interpersonal skills
- demonstrate intellectual curiosity, an entrepreneurial spirit and initiative
- reflect critically on ethical issues

The mathematics curriculum contributes to personal development in a number of ways. First, it demonstrates the central role of mathematics in a wide variety of career options. Second, it provides a vehicle by which students can learn to work on problems both cooperatively and independently. Finally, the study of mathematics will help develop the intellectual curiosity, persistence and habits of mind which will serve individuals as lifelong learners.

Problem Solving

Graduates will be able to use the strategies and processes needed to solve a wide variety of problems, including those requiring language, and mathematical and scientific concepts.

Graduates will be able, for example, to

- acquire, process and interpret information critically to make informed decisions
- use a variety of strategies and perspectives with flexibility and creativity for solving problems
- formulate tentative ideas and question their own assumptions and those of others
- solve problems individually and collaboratively
- identify, describe, formulate and reformulate problems
- frame and test hypotheses
- ask questions, observe relationships, make inferences and draw conclusions
- identify, describe and interpret different points of view and distinguish fact from opinion

Problem solving is a unifying idea within the mathematics curriculum. Through the study of mathematics, students will learn and apply a large variety of information processing and interpretation skills, problem-solving strategies and critical and creative thinking skills. Ultimately, problem solving is the primary reason for doing mathematics.

Technological Competence

Graduates will be able to use a variety of technologies, demonstrate an understanding of technological applications and apply appropriate technologies for solving problems.

Graduates will be able, for example, to

- locate, evaluate, adapt, create and share information, using a variety of sources and technologies
- demonstrate understanding of and use existing and developing technologies
- demonstrate understanding of the impact of technology on society
- demonstrate understanding of ethical issues related to the use of technology in a local and global context

The mathematics curriculum will contribute to the development of technological competence in several ways. Students should not only regularly use calculators, graphics calculators and computer software, but they should also learn when it is appropriate to use such technologies. As well, students will be open to the use of new technology and develop an appreciation of the impact of technology on the study of and nature of doing mathematics.

INTRODUCTION TO CURRICULUM OUTCOMES

VISION

The Atlantic Canada mathematics curriculum is shaped by a vision which fosters the development of mathematically literate students who can extend and apply their learning and who are effective participants in an increasingly technological society.

UNIFYING IDEAS

To help accomplish this vision and to contribute significantly to aiding students in meeting the essential graduation learnings for Atlantic Canada, the mathematics curriculum is focussed around four unifying ideas. These unifying ideas permeate the mathematics curriculum across topics and grades and reiterate the key curriculum standards identified by the National Council of Teachers of Mathematics.

Upon graduation from high school, all students will

- demonstrate confidence and competence in their ability to use problem-solving approaches to investigate and understand mathematical content, to use mathematics to solve both routine and non-routine problems from within and outside mathematics, to recognize multiple solutions to problems and to apply mathematical modelling to real-world problem situations
- be able to communicate mathematically: students must reflect on and clarify their mathematical thinking, formulate definitions and express generalizations, recognize multiple representations of concepts, express ideas orally and in writing, read mathematics with understanding, ask clarifying and extending questions, and appreciate the economy, power and elegance of mathematical notation
- be able to reason mathematically: students must be able to demonstrate logical reasoning skills that include making and testing conjectures, formulating counterexamples, judging the validity of arguments and constructing simple, valid arguments
- value mathematics: students will understand how mathematics and historical situations have interacted and how this has influenced their lives; they will be able to identify and use connections both within mathematics and between mathematics and other disciplines and see how these connections impact on their world

These unifying ideas influence all the key-stage curriculum outcomes which follow (see pp. 12-25) and which are further elaborated in detailed curriculum guides. The vision of mathematics education is that all students will work toward the same curricular outcomes. The key-stage curriculum outcomes are arranged by content strands (see pp. 10-11) which organize the skills and concepts of an appropriate body of mathematical knowledge for school students.

Problem Solving

"To solve a problem is to find a way where no way is known off-hand, to find a way out of a difficulty, to find a way around an obstacle, to attain a desired end, that is not immediately attainable, by appropriate means."

(G. Polya quoted by Krulik and Reys in the NCTM *Curriculum and Evaluation Standards*, p. 75)

Problem solving should be a major focus of all mathematics, given that the purpose of mathematics is to make sense of the world. As people encounter the world throughout their lives, they are faced with "problems" which require solutions if they are to meet their goals. Mathematical problem solving serves not only to answer questions raised in everyday life, but also to deal with issues from professions such as business and engineering, in other disciplines such as the physical and social sciences, and which extend and connect mathematics theory itself.

Problem-solving activities need to include an appropriate combination of situations from the relatively clearcut to the more complex and/or abstract. Students need to experience sufficient success in solving problems to gain confidence in doing mathematics, persevere in tasks that require extended effort and become encouraged to take intellectual risks when faced with novel situations. Problem solving must provide opportunities for students to explore alternate lines of approach and to appreciate that there may well be more than one possible outcome.

Ultimately, problem solving needs to be seen as not a distinct topic but rather as a process that must permeate the entire mathematics program. It will serve as a means to introduce new mathematical content, develop the understanding of concepts and facility with procedures and apply that which is already known. Problem-solving strategies, including the application of technological resources, should be regularly modelled by teachers and peers so that they become increasingly integrated by students as a basis for "doing mathematics."

"The curriculum must give students opportunities to solve problems that require them to work cooperatively, to use technology, to address relevant and interesting mathematical ideas, and to experience the power and usefulness of mathematics."
(NCTM *Curriculum and Evaluation Standards*, p. 75-76)

Communication

"Mathematics can be thought of as a language that must be meaningful if students are to communicate mathematically and apply mathematics productively."
(NCTM *Curriculum and Evaluation Standards*, p.6)

Communication plays a major role in the development of mathematical understanding. While talking often precedes writing, both verbal and written communication are of great value with respect to children constructing knowledge, learning alternative ways to think about ideas and clarifying their own thinking. Communication allows students to construct links between their informal notions and the abstract language and symbolism of mathematics; recognize the importance of common definitions; achieve deeper understandings of concepts and principles; and clarify, refine and consolidate their thinking.

Students need to be daily encouraged to discuss and write about mathematical ideas. They must engage in such activities as describing mathematical processes, explaining and differentiating among mathematical concepts, explaining their reasoning and evaluating the responses of others. Through the process of regular communication the use of the language and symbols of mathematics becomes natural.

As well as communicating to others mathematically, it is increasingly necessary that students read mathematics effectively. There is a vast difference between reading a work of fiction and reading a piece of mathematical writing, yet doing the latter well is necessary if print mathematical materials are to be valuable resources. Reading for mathematical understanding does not readily lend itself to skimming. Rather each part of mathematical statements must be carefully considered for meaning and readers must take full advantage of accompanying illustrative material.

"The ability to read, write, listen, think creatively, and communicate about problems will develop and deepen students' understanding of mathematics."
(NCTM *Curriculum and Evaluation Standards*, p.78)

Reasoning

"Mathematics is reasoning. One cannot do mathematics without reasoning."
(NCTM *Curriculum and Evaluation Standards*, p.29)

Because reasoning is fundamental to the knowing and doing of mathematics, it is important that reasoning be a regular focus of mathematical activity. Teachers should consistently model the use of mathematical reasoning and engage students in discussions and tasks designed to develop their capacities to reason mathematically.

The development of logical reasoning is tied to the intellectual and verbal development of a student. The development of the student's ability to reason is a process that grows out of many experiences with such activities as analyzing problems, making conjectures, gathering evidence and building arguments to support ideas. Students need to focus in numeric, spatial and algebraic contexts on such questions as: Are we sure? How can we be sure? What conditions could make you sure? What obstacles are there to being sure?

In the early grades, the development of mathematical reasoning ability can be fostered by often involving students in the kind of informal thinking, conjecturing and validating that helps students to see that mathematics makes sense. In later grades, as the depth and complexity of mathematical content is increased, these informal reasoning activities should be expanded to include more formal recognition of the interplay between conjecturing and inductive reasoning and the importance of deductive verification.

Reasoning is an integral part of the study of mathematics and demonstrations of reasoning should be openly valued. Students at all levels should be encouraged to justify their solutions, thinking processes and conjectures in a variety of ways.

"Conjecturing and demonstrating the logical validity of conjectures are the essence of the creative act of doing mathematics. To give more students access to mathematics as a powerful way of making sense of the world, it is essential that an emphasis on reasoning pervade all mathematical activity."
(NCTM *Curriculum and Evaluation Standards*, p.81)

Connections

"We can no longer afford to let mathematics remain an isolated discipline nor to permit continued fragmentation within the mathematics curriculum itself into isolated courses, separate topics, and disconnected bits of knowledge."
(Steen, Lynn Arthur, "Teaching Mathematics for Tomorrow's World," *Educational Leadership*, Sept. 1989, p. 18)

Teachers of mathematics should strive to make natural connections among mathematical concepts and representations, and to emphasize the natural connections that exist across curricula.

Inherent in this integrative focus is the potential for important educational benefits for both teachers and students which have strong foundations in recognized principles of learning and cognitive development. These benefits include, but are not limited to,

- the use of active, experiential learning
- an increase in student motivation and interest
- the teaching of mathematics in context and with relevance
- the development of higher order thinking skills
- the development of cooperative and collaborative interpersonal skills
- a strengthening of students' understanding of mathematics concepts and principles
- an understanding of the interdependent nature of the world
- the development of critical problem-solving skills

As well as highlighting connections among mathematical ideas and with other areas of the school curriculum, the mathematics program should also address connections with career areas and other aspects of the everyday world.

"It is important that children connect ideas both among and within areas of mathematics.... When mathematical ideas are also connected to everyday experiences, both in and out of school, children become aware of the usefulness of mathematics."
(NCTM *Curriculum and Evaluation Standards*, p. 32)

CURRICULUM OUTCOMES

STRANDS AND GENERAL CURRICULUM OUTCOMES

The outcomes for the mathematics curriculum are organized in terms of four content strands:

- number concepts/number and relationship operations
- patterns and relations
- shape and space
- data management and probability

One or two general curriculum outcomes (i.e., statements which identify what students are expected to know and be able to do upon completion of study in a curriculum area) are identified for each of these strands. The general curriculum outcomes (GCOs) are then further elaborated (pp. 12-25) in terms of key-stage curriculum outcomes (i.e., outcomes at the end of each of grades 3, 6, 9 and 12).

The content strands and general curriculum outcomes are detailed as follows:

Number Concepts/Number and Relationship Operations

- Students will demonstrate number sense and apply number theory concepts.
- Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Patterns and Relations

- Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

Shape and Space

- Students will demonstrate an understanding of and apply concepts and skills associated with measurement.
- Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

Data Management and Probability

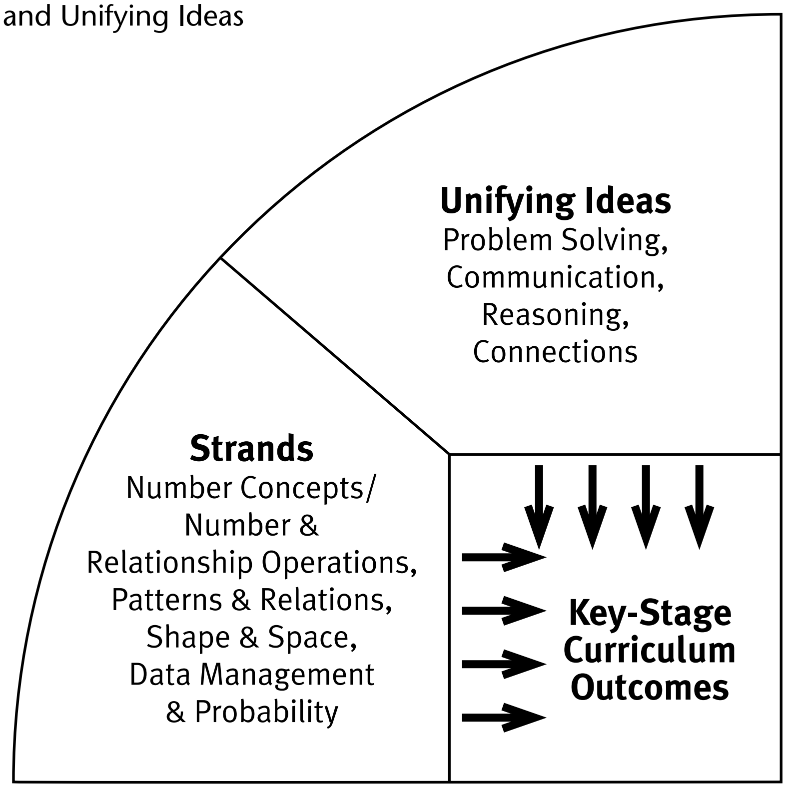
- Students will solve problems involving the collection, display and analysis of data.
- Students will represent and solve problems involving uncertainty.

It is critical that the unifying ideas outlined in the previous section (i.e., problem solving, communication, reasoning and connections) strongly influence, in fact permeate, the outcomes articulated for the content strands. As indicated in the diagram following, this integration of the strands and the unifying ideas takes place in the development of the key-stage curriculum outcomes.

It must be noted that, while the key-stage curriculum outcomes are intended as targets for all students, all students will not be expected to achieve them at a single level of performance. As well, there will be an additional small percentage of students who will see their outcomes significantly altered in individual educational programs.

FIGURE 2 – Integrating Strands and Unifying Ideas

The key-stage curriculum outcomes reflect the infusion of the unifying ideas into the content strands.



KEY-STAGE CURRICULUM OUTCOMES

GCO: Students will demonstrate number sense and apply number theory concepts.

Elaboration: Number sense includes understanding of number meanings, developing multiple relationships among numbers, recognizing the relative magnitudes of numbers, knowing the relative effect of operating on numbers and developing referents for measurement. Number theory concepts include such number principles as laws (e.g., commutative and distributive), factors and primes, number system characteristics (e.g., density), etc.

By the end of grade 3, students will be expected to	*By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to*
construct and communicate number meanings, and explore and apply estimation strategies, with respect to whole numbers	demonstrate an understanding of number meanings with respect to whole numbers, fractions and decimals
concretely explore common fractions and decimals in meaningful situations	explore integers, ratios and percents in common, meaningful situations
read and write whole numbers and demonstrate an understanding of place value (to four places)	read and write whole numbers and decimals and demonstrate an understanding of place value (to millions and to thousandths)
order whole numbers and represent them in multiple ways	order whole numbers, fractions and decimals and represent them in multiple ways
apply number theory concepts (e.g., place value pattern) in meaningful contexts with respect to whole numbers and commonly used fractions and decimals	apply number theory concepts (e.g., prime numbers, factors) in relevant situations with respect to whole numbers, fractions and decimals

GCO: Students will demonstrate number sense and apply number theory concepts.

Elaboration: Number sense includes understanding of number meanings, developing multiple relationships among numbers, recognizing the relative magnitudes of numbers, knowing the relative effect of operating on numbers and developing referents for measurement. Number theory concepts include such number principles as laws (e.g., commutative and distributive), factors and primes, number system characteristics (e.g., density), etc.

By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to	*By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to*
demonstrate an understanding of number meanings with respect to integers and rational and irrational numbers, and explore their use in meaningful situations	demonstrate an understanding of number meanings with respect to the real numbers
read, write and order integers, rational numbers and common irrational numbers represent numbers in multiple ways (including via exponents, ratios, percents and proportions) and apply appropriate representations to solve problems	order real numbers, represent them in multiple ways (including scientific notation) and apply appropriate representations to solve problems
apply number theory concepts in relevant situations and explain the interrelated structure of whole numbers, integers and rational numbers	demonstrate an understanding of the real number system and its subsystems by applying a variety of number theory concepts in relevant situations
	Some post-secondary intending students will be expected to explain and apply relationships among real and complex numbers

GCO: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration: Operation sense consists of recognizing situations in which a given operation would be useful, building awareness of models and the properties of an operation, seeing relationships among operations and acquiring insights into the effects of an operation on a pair of numbers. Operation principles and procedures would include such items as the effect of identity elements, computational strategies and mental mathematics.

By the end of grade 3, students will be expected to	*By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to*
demonstrate an understanding of the connection between relevant, concrete experiences and the mathematical language and symbolism of the four basic operations	model problem situations involving whole numbers and decimals by selecting appropriate operations and procedures
recognize and explain the relationships among the four basic operations	model problem situations involving the addition and subtraction of simple fractions
create and model problem situations involving whole numbers, using one or more of the four basic operations	explore algebraic situations informally
demonstrate proficiency with addition and subtraction facts	apply computational facts and procedures (algorithms) in a wide variety of problem situations involving whole numbers and decimals
apply computational facts and strategies with respect to the four basic operations and model addition and subtraction in situations involving whole numbers	
apply estimation techniques to predict, and justify the reasonableness of, results in relevant problem situations involving whole numbers	apply estimation techniques to predict, and justify the reasonableness of, results in relevant problem situations involving whole numbers and decimals
select and use appropriate computational techniques (including mental, paper-and-pencil and technological) in given situations	select and use appropriate computational techniques (including mental, paper-and-pencil and technological) in given situations

GCO: Students will demonstrate operation sense and apply operation principles and procedures in both numeric and algebraic situations.

Elaboration: Operation sense consists of recognizing situations in which a given operation would be useful, building awareness of models and the properties of an operation, seeing relationships among operations and acquiring insights into the effects of an operation on a pair of numbers. Operation principles and procedures would include such items as the effect of identity elements, computational strategies and mental mathematics.

By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to	*By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to*
explore and explain, using physical models, the connections between arithmetic and algebraic operations model problem situations involving rational numbers and integers	explain how algebraic and arithmetic operations are related, use them in problem-solving situations, and explain and demonstrate the power of mathematical symbolism
apply computational procedures (algorithms) in a wide variety of problem situations involving fractions, ratios, percents, proportions, integers and exponents apply operations to algebraic expressions to represent and solve relevant problems	derive, analyze and apply computational procedures (algorithms) in situations involving all representations of real numbers derive, analyze and apply algebraic procedures (including those involving algebraic expressions and matrices) in problem situations
apply estimation techniques to predict, and justify the reasonableness of, results in relevant problem situations involving rational numbers and integers select and use appropriate computational techniques in given situations and justify the choice	apply estimation techniques to predict, and justify the reasonableness of, results in relevant problem situations involving real numbers
	Some post-secondary intending students will be expected to apply operations on complex numbers to solve problems

GCO: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

Elaboration: Patterns and relationships run the gamut from number patterns and those made from concrete materials to polynomial and exponential functions. The representation of patterns and relationships will take on multiple forms, including sequences, tables, graphs and equations, and these representations will be applied as appropriate in a wide variety of relevant situations.

By the end of grade 3, students will be expected to	*By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to*
recognize, describe, extend and create patterns and sequences in a variety of mathematical and real-world contexts (e.g., geometric, numeric and measurement) use patterns to solve problems	describe, extend and create a wide variety of patterns and relationships to model and solve problems involving real-world situations and mathematical concepts explore how a change in one quantity in a relationship affects another
represent mathematical patterns and relationships in informal ways, including via open sentences (i.e., statements with missing addends)	represent mathematical patterns and relationships in a variety of ways (including rules, tables and one- and two-dimensional graphs)
	solve linear equations using informal, non-algebraic methods

GCO: Students will explore, recognize, represent and apply patterns and relationships, both informally and formally.

Elaboration: Patterns and relationships run the gamut from number patterns and those made from concrete materials to polynomial and exponential functions. The representation of patterns and relationships will take on multiple forms, including sequences, tables, graphs and equations, and these representations will be applied as appropriate in a wide variety of relevant situations.

By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to	*By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to*
analyze, generalize and create patterns and relationships to model and solve real-world and mathematical problem situations analyze functional relationships to explain how the change in one quantity results in a change in another	model real-world problems using functions, equations, inequalities and discrete structures
represent patterns and relationships in multiple ways (including the use of algebraic expressions, equations, inequalities and exponents) explain the connections among algebraic and non-algebraic representations of patterns and relationships	represent functional relationships in multiple ways (e.g., written descriptions, tables, equations and graphs) and describe connections among these representations interpret algebraic equations and inequalities geometrically and geometric relationships algebraically
apply algebraic methods to solve linear equations and inequalities and investigate non-linear equations	solve problems involving relationships, using graphing technology as well as paper-and-pencil techniques analyze and explain the behaviours, transformations and general properties of types of equations and relations perform operations on and between functions
	Some post-secondary intending students will be expected to describe and explore the concept of continuity of a function investigate limiting processes by examining infinite sequences and series make connections among trigonometric functions, polar coordinates, complex numbers and series

GCO: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Elaboration: Concepts and skills associated with measurement include making direct measurements, using appropriate measurement units and using formulas (e.g., surface area, Pythagorean Theorem) a/o procedures (e.g., proportions) to determine measurements indirectly.

By the end of grade 3, students will be expected to	*By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to*
measure and understand basic concepts and attributes of length, capacity, mass, area and time	extend understanding of measurement concepts and attributes to include volume, temperature, perimeter and angle
identify and use non-standard and standard units of measurement and appreciate their role in communication	communicate using standard units, understand the relationship among commonly used SI units (e.g., mm, cm, m, km) and select appropriate units in given situations
estimate and determine measurements in every-day problem situations and develop a sense of the relative size of units	estimate and apply measurement concepts and skills in relevant problem situations and select and use appropriate tools and units
	develop and apply rules and procedures for deter-mining measures (using concrete and graphing models)

GCO: Students will demonstrate an understanding of and apply concepts and skills associated with measurement.

Elaboration: Concepts and skills associated with measurement include making direct measurements, using appropriate measurement units and using formulas (e.g., surface area, Pythagorean Theorem) a/o procedures (e.g., proportions) to determine measurements indirectly.

By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to	*By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to*
demonstrate an understanding of the concept of rates; use direct and indirect measurements to describe and make comparisons and read and interpret scales; and describe how a change in one measurement affects other, indirect measurements	measure quantities indirectly, using techniques of algebra, geometry and trigonometry
communicate using a full range of SI units (e.g., mm, cm, dm, m, hm, Dm, km) and select appropriate units in given situations	
estimate and apply measurement concepts in relevant problem situations and use tools and units which reflect an appropriate degree of accuracy	determine measurements in a wide variety of problem situations and determine specified degrees of precision, accuracy and error of measurements
develop and apply a wide range of measurement formulas and procedures (including indirect measures)	apply measurement formulas and procedures in a wide variety of contexts
	Some post-secondary intending students will be expected to demonstrate an understanding of the meaning of area under a curve

GCO: Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

Elaboration: Spatial sense is an intuitive feel for one's surroundings and the objects in them and is characterized by such geometric relationships as i) the direction, orientation and perspectives of objects in space, ii) the relative shapes and sizes of figures and objects and iii) how a change in shape relates to a change in size. Geometric concepts, properties and relationships are illustrated by such examples as the concept of area, the property that a square maximizes area for rectangles of a given perimeter, and the relationships among angles formed by a transversal intersecting parallel lines.

By the end of grade 3, students will be expected to	*By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to*
explore and experiment with geometric shapes and relationships (including the orientation and perspectives of objects)	identify, draw and build physical models of geometric figures
describe, model, draw and classify 2- and 3-D figures and shapes	describe, model and compare 2- and 3-D figures and shapes, explore their properties and classify them in alternative ways
investigate and predict the results of combining, subdividing and transforming shapes	investigate and predict the results of transformations and begin to use them to compare shapes and explain geometric concepts (e.g., symmetry and similarity)
relate geometric ideas to number and measurement ideas and recognize and apply geometric principles in real-world situations	solve problems using geometric relationships and spatial reasoning

GCO: Students will demonstrate spatial sense and apply geometric concepts, properties and relationships.

Elaboration: Spatial sense is an intuitive feel for one's surroundings and the objects in them and is characterized by such geometric relationships as i) the direction, orientation and perspectives of objects in space, ii) the relative shapes and sizes of figures and objects and iii) how a change in shape relates to a change in size. Geometric concepts, properties and relationships are illustrated by such examples as the concept of area, the property that a square maximizes area for rectangles of a given perimeter, and the relationships among angles formed by a transversal intersecting parallel lines.

By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to	*By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to*
construct and analyze 2- and 3-D models, using a variety of materials and tools	extend spatial sense in a variety of mathematical contexts
compare and classify geometric figures, understand and apply geometric properties and relationships, and represent geometric figures via coordinates	interpret and classify geometric figures, translate between synthetic (Euclidean) and coordinate representations, and apply geometric properties and relationships
develop and analyze the properties of transformations and use them to identify relationships involving geometric figures	analyze and apply Euclidean transformations, including representing and applying translations as vectors
represent and solve abstract and real-world problems in terms of 2- and 3-D geometric models	represent problem situations with geometric models (including the use of trigonometric ratios and coordinate geometry) and apply properties of figures
draw inferences, deduce properties and make logical deductions in synthetic (Euclidean) and transformational geometric situations	make and test conjectures about, and deduce properties of and relationships between, 2- and 3-D figures in multiple contexts demonstrate an understanding of the operation of axiomatic systems and the connections among reasoning, justification and proof
	Some post-secondary intending students will be expected to represent and apply vectors in three dimensions algebraically and geometrically explore and apply, using multiple representations, circles, ellipses and parabolas and, in 3-D, spheres and ellipsoids

GCO: Students will solve problems involving the collection, display and analysis of data.

Elaboration: The collection, display and analysis of data involves i) attention to sampling procedures and issues, ii) recording and organizing collected data, iii) choosing and creating appropriate data displays, iv) analyzing data displays in terms of broad principles (e.g., display bias) and via statistical measures (e.g., mean) and v) formulating and evaluating statistical arguments.

By the end of grade 3, students will be expected to	*By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to*
collect, record, organize and describe relevant data	collect, organize and describe relevant data in multiple ways
construct concrete and pictorial displays of relevant data	construct a variety of data displays (including tables, charts and graphs) and consider their relative appropriateness
read and interpret displays of relevant data	read, interpret, and make and modify predictions from displays of relevant data
	develop and apply measures of central tendency (mean, median and mode)
generate questions, develop and modify predictions and implement plans with respect to data analysis	formulate and solve simple problems (both real-world and from other academic disciplines) that involve the collection, display and analysis of data and explain conclusions which may be drawn

GCO: Students will solve problems involving the collection, display and analysis of data.

Elaboration: The collection, display and analysis of data involves i) attention to sampling procedures and issues, ii) recording and organizing collected data, iii) choosing and creating appropriate data displays, iv) analyzing data displays in terms of broad principles (e.g., display bias) and via statistical measures (e.g., mean) and v) formulating and evaluating statistical arguments.

By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to	*By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to*
be aware of sampling issues and understand procedures with respect to collecting data	understand sampling issues and their role with respect to statistical claims
construct various data displays (both manually and via technology) and decide which is/are most appropriate	extend construction (both manually and via appropriate technology) of a wide variety of data displays
draw inferences and make predictions from a variety of displays of real-world data (including via curve-fitting with respect to scatterplots)	use curve fitting to determine the relationship between, and make predictions from, sets of data and be aware of bias in the interpretation of results
determine, and apply as appropriate, measures of central tendency and dispersion (e.g., range)	determine, interpret and apply as appropriate a wide variety of statistical measures and distributions
demonstrate an appreciation of statistics as a decision-making tool by formulating and solving relevant problems (e.g., projects with respect to current issues a/o other academic disciplines) make convincing statistical arguments and evaluate those of others	design and conduct relevant statistical experiments (e.g., projects with respect to current issues, career applications, a/o other disciplines) and analyze and communicate the results using a range of statistical arguments
	Some post-secondary intending students will be expected to test hypotheses using appropriate statistics

GCO: Students will represent and solve problems involving uncertainty.

Elaboration: Representing and solving problems involving uncertainty entails i) determining probabilities by conducting experiments a/o making theoretical calculations, ii) designing simulations to determine probabilities in situations which do not lend themselves to direct experiment and iii) analyzing problem situations so as to decide how best to determine probabilities.

By the end of grade 3, students will be expected to	*By the end of grade 6, students will have achieved the outcomes for entry-grade 3 and will also be expected to*
conduct informal investigations of chance and estimate probabilities with respect to games and other simple, everyday situations	explore, interpret and make conjectures about everyday probability situations by estimating probabilities, conducting experiments, beginning to construct and conduct simulations, and analyzing claims which they see and hear
	determine theoretical probabilities using simple counting techniques
	demonstrate an understanding of the relationship between the numerical expression describing a probability and the events which give rise to the numbers

GCO: Students will represent and solve problems involving uncertainty.

Elaboration: Representing and solving problems involving uncertainty entails i) determining probabilities by conducting experiments a/o making theoretical calculations, ii) designing simulations to determine probabilities in situations which do not lend themselves to direct experiment and iii) analyzing problem situations so as to decide how best to determine probabilities.

By the end of grade 9, students will have achieved the outcomes for entry-grade 6 and will also be expected to	*By the end of grade 12, students will have achieved the outcomes for entry-grade 9 and will also be expected to*
make predictions regarding and design and carry out probability experiments and simulations, in relation to a variety of real-world situations	design and conduct experiments a/o simulations to model and solve a wide variety of relevant probability problems, and interpret and judge the probabilistic arguments of others
derive theoretical probabilities, using a range of formal and informal techniques	build and apply formal concepts and techniques of theoretical probability (including the use of permutations and combinations as counting techniques)
determine and compare experimental and theoretical results	understand the differences among, and relative merits of, theoretical, experimental and simulation techniques
relate a variety of numerical expressions (ratios, fractions, decimals, percents) to the corresponding experimental or simulation situation	relate probability and statistical situations
	Some post-secondary intending students will be expected to create and interpret discrete and continuous probability distributions and apply them in real-world situations

Contexts for Learning and Teaching

LEARNING AND TEACHING MATHEMATICS

PRINCIPLES OF LEARNING AND TEACHING MATHEMATICS

The following principles of learning and teaching mathematics are based on current research and best practice. They are intended to guide and support decisions relating to the teaching and learning process, including curriculum and classroom organization, assessment and reporting.

- ***Students should be viewed as individuals who develop and learn via different styles and at different rates.***

  Students' strengths, needs and backgrounds must be recognized and addressed through a variety of teaching methods. All students should be consistently and appropriately challenged. They must be provided the time, resources and feedback necessary to reach their potential.

- ***Students benefit, both socially and intellectually, from a variety of learning experiences, both independent and in collaboration with others.***

  Instructional settings should include varied learning configurations—whole class, small group, pairs and individual—as well as varied activities, including investigations, discussions, oral presentations and projects.

- ***Students learn best in an environment which supports exploration, investigation, critical and creative thinking, risk taking, reflection and other higher order thinking skills.***

  Learning mathematics is a constructive rather than a passive activity. Students need to learn mathematics in a climate in which they feel free to explore, knowing that their efforts, ideas and approaches will be valued by both their teacher and their classmates. Students should be encouraged to raise questions and investigate answers, thereby influencing the direction of their learning.

- ***Students learn best when ideas are approached in a variety of ways.***

  Multiple representations of mathematical ideas enhance student understanding. (For example, relations may be readily represented both graphically and via equations or inequations.) As well, students should be encouraged to represent their own understandings in various ways.

- ***Students learn best in an environment that nurtures positive attitudes, sustained effort, self-discipline and the development of an appropriate degree of autonomy.***

  Students should be praised for effort, original thought and perseverance. If these are qualities which the students are to perceive as important, they must be reflected as well in assessment practices.

- ***Students' learning is most effective when standards of expectation are made clear and explicit.***

  Clearly stated expectations enhance academic achievement and feelings of self-esteem. Well-articulated outcomes allow students to monitor their daily work and improve their learning.

RESOURCE-BASED LEARNING

As students and schools enter the 21st century, they find themselves in an era of rapid change and rapidly expanding knowledge. It is no longer adequate or realistic for students to acquire a select body of knowledge and expect it to meet their needs as citizens of the next century. The need for lifelong learning is shifting the emphasis from a dependence on the 'what' of learning to the 'how' of learning—today's students must 'learn how to learn.'

Resource-based learning is a philosophy which stresses a shift from the use of a single resource in the classroom to the use of a wide variety of print and non-print resources. Resources include multi-media, telecommunication and human, as well as a wide variety of print resources, both in the classroom and in libraries/ resource centres. This philosophy of learning complements the approach to mathematical instruction identified in this document and is characterized by

- students actively participating in their learning
- teachers acting as facilitators of learning, continuously guiding, monitoring and evaluating student progress
- varying locations for learning
- learning experiences based on curriculum outcomes
- learning strategies and skills identified and taught within the context of relevant and meaningful units of study
- teachers working together to facilitate resource-based learning across grade levels and subject areas

Resource-based learning has many advantages. With students at the centre of the instructional process, they will

- acquire skills and attitudes necessary for independent, lifelong learning; they learn how to learn, one of the fundamental aims of education
- interact in group work, sharing and participating in a variety of situations
- think critically and creatively, experimenting and taking risks as they become independent and collaborative problem-solvers and decision-makers
- make choices and accept responsibility for these choices, thereby making learning more relevant and personal

BALANCING PROCEDURAL AND CONCEPTUAL KNOWLEDGE

Many view mathematics as a set of rules and procedures. Fundamentally, however, mathematics is a set of ideas. The intent of this curriculum is to ensure that students understand these ideas, not just master the rules and procedures. The re-creation, transfer and application of ideas is enhanced when students assimilate them into a conceptual framework. At the same time, it is essential that students accomplish a certain level of skill proficiency so that they have the tools to solve interesting and relevant problems.

Students should be encouraged to invent and examine various strategies that might help them develop skills. This suggests that the teacher encourage diverse approaches rather than requiring a single "best" approach for all students to use. For example, some students may solve the equation $6x+36=90$ by first subtracting 36 from each side and then dividing by 6. Others might initially divide both sides of the equation by 6 and then subtract 6 from each side. Generally, those who are most successful in mathematics are those who use different strategies in different situations.

Practice of skills is usually more effective if the practice arises in meaningful contexts. For example, rather than requiring students to complete a series of unrelated exercises, teachers could structure game formats in which the fast pace encourages proficiency with skills, as well as providing situations involving related questions so that patterns and underlying ideas become clearer.

More importantly, skills must be embedded in, and arise from, conceptual situations as much as possible. It is of little value to a student to be able to use a tree diagram if he or she does not understand that the tree diagram is valuable in certain types of counting situations. Thus, as much attention must be paid to when such diagrams are useful as to how to create them.

Mathematics is much more than procedures, with a bit of conceptual learning thrown in. Curricula and instructional practice for the future must adjust this balance to place greater emphasis on conceptual understanding.

ESTIMATION AND MENTAL COMPUTATION

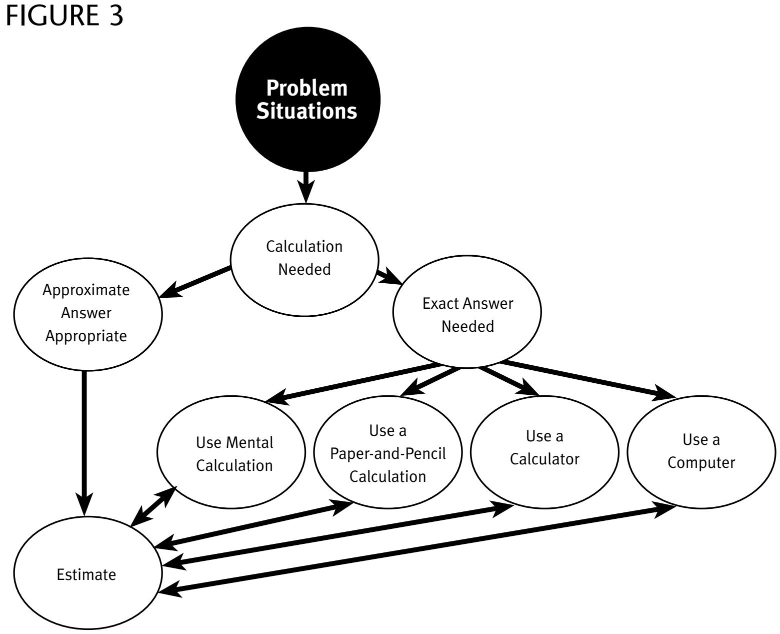
The NCTM *Curriculum and Evaluation Standards*' view of computation extends far beyond the traditional paper-and-pencil algorithms. When faced with a numerical problem, a decision about calculation procedures has to be made. As illustrated in figure 3 (NCTM *Curriculum and Evaluation Standards*, p. 9), students should have a repertoire of computational techniques available and must learn to select an appropriate technique in each given situation.

Both estimation and mental computation (more often referred to as "mental math") are done "in one's head" and have been deemed important skills by the NCTM. Computing mentally to find approximate or exact answers is frequently the most appropriate and useful of all methods. There are specific strategies and algorithms for mental computation that must be taught and practised regularly.

Estimation and mental math are not topics that can be isolated as a unit of instruction; they must be integrated throughout the study of mathematics. It is important to remember that

- skill in mental computation takes time to develop
- mental computation must be practised regularly
- mental computation is best developed in context
- a variety of strategies for one computation should be encouraged and shared
- students need to understand why particular procedures work; they should not be taught computational tricks without understanding what they are doing
- a thorough understanding of, and facility with, computation is a valuable prerequisite for eventual use in algebra

FIGURE 3



HOMEWORK

Homework is an essential component of the mathematics program, as it extends the opportunity for students to think mathematically and to reflect on ideas explored during class time. The provision of this additional time for reflection and practice plays a valuable role in helping students to consolidate their learning.

Traditionally, homework has meant completing ten to twenty drill and practice questions relating to the procedure taught in a given day. With the increased emphasis on problem solving, conceptual understanding and mathematical reasoning, however, it is important that homework assignments change accordingly. More assignments involving problem solving, mathematical investigations, written explanations and reflections, and data collection should reduce some of the basic practice exercises given in isolation. In fact, a good problem can sometimes accomplish more than many drill-oriented exercises on a topic. For example, instead of a number of separate questions, problems such as the following might be considered:

- Write as many addition and subtraction sentences as you can that produce the answer of 10.
- Predict how often you will get "tails" if you flip a penny fifty times. Try it. Would it differ if you used a loonie? Check to see and write about your findings.
- A bike lock has four different numbers in the combination. The sum of the last two numbers is the first number. The sum of all the numbers is 15. What might the combination be?
- Why does trying to obtain the tangent of 270° from a calculator produce an error message?

As is the case in designing all types of homework, the needs of the students and the purpose of the assignment will dictate the nature of the questions included. Homework need not be limited to reinforcing learning; it provides an excellent opportunity to revisit topics explored previously and to introduce new topics.

Homework provides an effective way to communicate with parents and provides parents an opportunity to be actively involved in their child's learning. By ensuring that assignments model classroom instruction and sometimes require parental input, a parent will have a clearer understanding of the mathematics curriculum and the progress of his or her child in relationship to it. As Van de Walle (1994, p. 454) suggests, homework can serve as a parent's window to the classroom.

THE LEARNING ENVIRONMENT

The call for change in mathematics instruction is characterized by not only *what* mathematics is taught but also the *manner* in which it is taught. What students learn is fundamentally connected to how they learn it. A classroom environment in which optimal learning can take place must be established and maintained; it forms the learning foundation upon which skills and attitudes are constructed.

There are four unifying ideas that are central to the overall view of mathematics, as outlined in the NCTM *Curriculum and Evaluation Standards* (1989). They address problem solving, reasoning, communication and connections, and are common across all grade levels (entry-grade 12). In order to meet the requirements of these ideas, classrooms must look very different from those of the past.

The NCTM *Professional Standards* (p.3) lists five major shifts in the environment of mathematics classrooms (as shown in the table below).

Structurally the classroom environment will need to provide the flexibility to accommodate varying student needs and learning styles and to facilitate the active involvement of students in the development of mathematical concepts and skills. Movement among whole class, small group and independent experiences will be essential to provide all important contexts for learning.

As well, for students to develop a belief in themselves as mathematical thinkers, they must learn in a climate in which there is

- a genuine respect for others' ideas
- a valuing of reason and sense-making
- pacing and timing that allow students to puzzle and to think
- the forging of a social and intellectual community

Students, teachers and parents each play a significant role in creating this learning environment. Students have a responsibility with regard to participation, behaviour and work ethic. Teachers convey their attitude toward, and their value of, mathematics through their classroom presentation, responses to students' ideas and solutions and in their assessment practices. Positive attitudes of parents encourage students to pursue and persist at studies of mathematics. The entire climate or environment must be conducive to the fostering of mathematically-thinking students.

Summary of Shifts in Instructional Practices

Moving away from:	**Toward:**
classrooms as collections of individuals	classrooms as mathematical communities
the teacher as the sole authority for right answers	logic and mathematical evidence as verification
primarily memorizing procedures	mathematical reasoning
an emphasis on mechanistic answer finding	conjecturing, inventing and problem solving
treating mathematics as a body of isolated concepts and procedures	connecting mathematics, its ideas and its applications

DIVERSITY IN STUDENT NEEDS

Every classroom comprises students at many different cognitive levels. Rather than choosing a certain level at which to teach, a teacher is responsible for tailoring instruction to reach as many of these students as possible. In general, this may be accomplished by assigning different tasks to different students or assigning the same open-ended task to most students.

Sometimes it is appropriate for a teacher to group students by interest or ability, assigning them different tasks in order to best meet their needs. These groupings may last anywhere from minutes to semesters but should be designed to help all students, whether strong, weak or average, to reach their highest potential.

There are other times when an appropriately open-ended task can be valuable to a broad spectrum of students. For example, asking students to make up an equation for which the answer is 5 allows some students to make up very simple equations while others can design more complex ones. The different equations constructed can become the basis for a very rich lesson, from which all students come away with a better understanding of what the solution to an equation really means.

Entry - grade 12 mathematics teachers must be provided with resource materials that include many open-ended tasks from which to choose. In addition, they need access to a broad spectrum of resources which focus on the needs of particular segments of the student population.

COGNITIVE DEVELOPMENT

To match instruction to the student's level of intellectual maturity, teachers need to be familiar with the stages of cognitive development, the student's ability to think mathematically and the student's preferred learning style. While there are general patterns of development as students move through a number of age spans, it must be emphasized that these stages are simply reference points for teachers. *Not all students move through these stages at the same rate, nor is any given student necessarily at the same stage in all areas within the mathematics program* (e.g., number concepts vs. spatial sense). It is therefore important for teachers at all levels to build flexibility into their programs to help address individual developmental variations.

Primary/Elementary (Ages Five to Ten)

Students initially learn most effectively through first-hand experience. In school they need time and space for active exploration. As their language fluency grows, so does their ability to represent their thoughts symbolically through writing, drawing, graphing and making models. Teachers should ensure that students experiment with a variety of ways to represent their knowledge and understanding.

Students use direct experience, objects and visual aids to help understanding. First-hand experiences are necessary, since students better understand symbolism if they have had adequate, meaningful experiences in the concrete and pictorial modes.

Students will begin to think about their own thinking and to develop more sophisticated strategies for learning and for remembering what they have learned.

"[At this stage] a developmentally-appropriate curriculum encourages the exploration of a wide variety of mathematical ideas in such a way that children retain their enjoyment of, and curiosity about, mathematics. It incorporates real-world contexts, children's experiences, and children's language in developing ideas. It recognizes

that children need considerable time to construct sound understandings and develop the ability to reason and communicate mathematically. It looks beyond what children appear to know to determine how they think about ideas. It provides repeated contact with important ideas in varying contexts throughout the year and from year to year."
(NCTM *Curriculum and Evaluation Standards*, p.16)

Middle Level/Junior High (Ages Ten to Fourteen)

During these years many students begin to think in abstractions. They are better able to understand the symbolic nature of mathematics and to use symbols to represent number relations and other mathematical abstractions. It should be noted that, although they are beginning to develop the ability to "manipulate" thoughts and ideas, they still need hands-on experiences. The way in which students process information leads to a greater success in solving abstract problems. Accumulated knowledge, combined with logical conceptual connections, allows solutions to multi-stage problems.

Another trend at this time is the development of hypothetical thinking and appreciation of the multiple possibilities that are presented by given circumstances. It is important to respect different ways of presenting ideas. Positive encouragement in a low-risk environment is required in order for students at this age to develop their talents.

"As vast changes occur in their intellectual, psychological, social, and physical development, students [in this stage] begin to develop their abilities to think and reason more abstractly. Throughout this period, however, concrete experiences should continue to provide the means by which they construct knowledge. From these experiences they abstract more complex meanings and ideas. The use of language, both written and oral, helps students clarify their thinking and report their observations as they form and verify their mathematical ideas."
(NCTM *Curriculum and Evaluation Standards*, p.68)

High School (Ages Fourteen to Nineteen)

During these years, thought processes become more precise and analytical, while interests become more specific and specialized.

Students may use abstract rules to solve problems but will need assistance and experience in recognizing contexts for the application of such rules. It is important to note that the ability to apply formal operational skills varies according to the degree of experience in an area; therefore, students need active involvement in constructing and applying mathematical ideas in meaningful contexts. Also, during these years students often prefer to do an in-depth investigation of an area of interest and choice.

As students continue to refine their thinking abilities, they demonstrate greater awareness of the complexity of issues and may reject simplistic explanations. Increased life experience provides more and new opportunities for refinement of previously-learned reasoning and thinking skills. Students develop the ability to move freely between the concrete and the abstract, yet still need instructional strategies providing both.

"The role of students in the learning process…should shift in preparation for their entrance into the work force or higher education. Experiences designed to foster continued intellectual curiosity and increasing independence should encourage students to become self-directed learners who routinely engage in constructing, symbolizing, applying, and generalizing mathematical ideas… [Further,] teachers and students [should] become natural partners in developing mathematical ideas and solving mathematical problems."
(NCTM *Curriculum and Evaluation Standards*, p.128)

GENDER AND CULTURAL EQUITY

The mathematics curriculum and mathematics instruction must be designed to equally empower both male and female students, as well as members of all cultural backgrounds. Ultimately, this should mean that not only should enrolments of students of both genders and various cultural backgrounds in public school mathematics courses reflect numbers in society, but also that representative numbers of both genders and the various cultural backgrounds should move on to successful post-secondary studies and careers in mathematics and mathematics-related areas.

Two obvious goals for mathematics curriculum and mathematics instruction which will help facilitate gender and cultural equity are i) to portray the historical development of mathematics as involving both men and women of a wide variety of cultures and ii) to ensure that the instructional environment is uniformly supportive of members of both genders and all cultural groups.

The first of these goals might be achieved by employing such instructional strategies as

- examining the lives of associated mathematicians (including men and women of various backgrounds) when introducing new mathematical topics in the classroom
- including the study of the lives of mathematicians such as Sophie Germain and Subrahmanyan Chandrasekhar as potential topics for individual research

Strategies designed to assist in the accomplishment of the second goal include

- using curriculum materials which present a positive image of mathematics and mathematicians
- employing an appropriate instructional balance between individual and group work
- ensuring equitable sharing of classroom resources (including teacher attention and support)
- monitoring classroom instruction for teacher bias
- facilitating contact between students and those working in mathematical professions
- reinforcing the belief that “all students can do mathematics”

ASSESSING AND EVALUATING STUDENT LEARNING

Assessment
is the systematic process of gathering information on student learning

Evaluation
is the process of analyzing, reflecting upon and summarizing assessment information and making judgments or decisions based upon the information gathered.

Assessment and evaluation are essential components of teaching and learning in mathematics. Without effective assessment and evaluation it is impossible to know whether students have learned, whether teaching has been effective or how best to address student learning needs. The quality of the assessment and evaluation in the educational process has a profound and well-established link to student performance. Research consistently shows that regular monitoring and feedback are essential to improving student learning. What is assessed and evaluated, how it is assessed and evaluated and how results are communicated send clear messages to students and others about what is really valued—what is worth learning, how it should be learned, what elements of quality are considered most important and how well students are expected to perform.

Teacher-developed assessments and evaluations have a wide variety of uses, such as

- providing feedback to improve student learning
- determining if curriculum outcomes have been achieved
- certifying that students have achieved certain levels of performance
- setting goals for future student learning
- communicating with parents about their children's learning
- providing information to teachers on the effectiveness of their teaching, the program and the learning environment
- meeting the needs of guidance and administration

ASSESSMENT

To determine how well students are learning, assessment strategies have to be designed to systematically gather information on the achievement of the curriculum outcomes. In planning assessments, teachers should use a broad range of strategies in an appropriate balance to give students multiple opportunities to demonstrate their knowledge, skills and attitudes. Many types of assessment strategies can be used to gather such information, including, but not limited to,

- formal and informal observations
- work samples
- anecdotal records
- conferences
- teacher-made and other tests
- portfolios
- learning journals
- questioning
- performance assessment
- peer- and self-assessment

EVALUATION

Evaluation involves teachers and others in analyzing and reflecting upon information about student learning gathered in a variety of ways. This process requires

- developing clear criteria and guidelines for assigning marks or grades to student work
- synthesizing information from multiple sources
- weighing and balancing all available information
- using a high level of professional judgment in making decisions based upon that information

REPORTING

Reporting on student learning should focus on the extent to which students have achieved the curriculum outcomes. Reporting involves communicating the summary and interpretation of information about student learning to various audiences who require it. Teachers have a special responsibility to explain accurately what progress students have made in their learning and to respond to parent and student inquiries about learning.

Narrative reports on progress and achievement can provide information on student learning which letter or number grades alone cannot. Such reports might, for example, suggest ways in which students can improve their learning and identify ways in which teachers and parents can best provide support.

Effective communication with parents regarding their children's progress is essential in fostering successful home-school partnerships. The report card is one means of reporting individual student progress. Other means include the use of conferences, notes and phone calls.

GUIDING PRINCIPLES

In order to provide accurate, useful information about the achievement and instructional needs of students, certain guiding principles for the development, administration and use of assessments must be followed. *Principles for Fair Student Assessment Practices for Education in Canada* (1993) articulates five fundamental assessment principles as follows:

- Assessment methods should be appropriate for and compatible with the purpose and context of the assessment.
- Students should be provided with sufficient opportunity to demonstrate the knowledge, skills, attitudes or behaviours being assessed.
- Procedures for judging or scoring student performance should be appropriate for the assessment method used and be consistently applied and monitored.
- Procedures for summarizing and interpreting assessment results should yield accurate and informative representations of a student's performance in relation to the curriculum outcomes for the reporting period.
- Assessment reports should be clear, accurate and of practical value to the audience for whom they are intended.

These principles highlight the need for assessment which ensures that

- the best interests of the student are paramount
- assessment informs teaching and promotes learning
- assessment is an integral and ongoing part of the learning process and is clearly related to the curriculum outcomes
- assessment is fair and equitable to all students and involves multiple sources of information

While assessments may be used for different purposes and audiences, all assessments must give each student optimal opportunity to demonstrate what he/she knows and can do.

ASSESSING STUDENT LEARNING IN THE MATHEMATICS CLASSROOM

Student Assessment

"Testing to assign grades is one of the most common forms of evaluation. But assessment is a much broader and basic task, one designed to determine what students know and how they think about mathematics."
(NCTM *Curriculum and Evaluation Standards*, p. 203)

Effective assessment of student learning in mathematics provides educators with important information about the instructional needs of students and about student achievement of the curriculum outcomes. NCTM has published *Assessment Standards for School Mathematics* which may be used as a guide to enable teachers to match assessment practice with the Atlantic Canada vision for mathematics. These principles provide a basis for best practice and are consistent with the *Principles for Fair Student Assessment Practices for Education in Canada*.

Alignment with Curriculum

Tasks and methods for assessing students' learning must be aligned with the curriculum outcomes and instructional practice. An assessment program that mirrors instruction and is interwoven with it is vital for a clear, reliable and valid picture of how students are progressing and of how well instruction meets a student's needs.

Assessment instruments chosen need to reflect the relative emphases given in instruction to topics and to processes. For example, an effective program assesses the student's progress with the skills and concepts that are stressed in a particular instructional unit, and it includes ongoing assessment of the process components—problem solving, communication, mathematical reasoning and making mathematical connections—to the extent that these processes are part of the instructional unit.

For assessment to be aligned with instruction, it is necessary that assessment tasks involve the use of the same materials and technologies used regularly in the classroom. The routine use of calculators, computers and manipulatives is integral to the success of this curriculum; therefore, assessment tasks incorporating the use of these materials and technologies are necessary.

EXTERNAL ASSESSMENT

Administration of externally prepared assessments is on a large scale in comparison to classroom assessments and often involves hundreds, sometimes thousands, of students, allowing for use of results at the provincial, district and/or school levels. Depending on the comprehensiveness of the assessment, information can be used for all of the same purposes as classroom-based assessment, but it can also serve additional administrative and accountability purposes such as for admissions, placement, student certification, educational diagnosis and program evaluation. External assessments offer common standards for assessment and for administration, scoring and reporting which allow for comparison of results over time.

As part of the regional agenda, development of external assessments in the core curriculum areas is being undertaken. Generally, external assessment includes assessments prepared by departments of education, national and international assessment groups, publishers and research groups. Each provincial department of education makes decisions on whether or not to administer external assessments.

PROGRAM AND SYSTEM EVALUATION

The results from both external and internal assessments of student achievement can be used to varying degrees for program and system evaluation. External assessment results, however, are more comparable across various groups and are therefore more commonly the basis for these types of evaluations.

In essence, the main difference between student evaluation and program and system evaluation is in how the results are used. In program evaluation marks or scores for individual students are not the primary focus of the assessment—it is the effectiveness of the program that is evaluated, and the results are used to show the extent to which the many outcomes of the program are achieved.

When results are used for system evaluation, the focus is on how the various levels and groups within the system, such as classrooms, schools, districts and so on, are achieving the intended outcomes. In many ways student and program evaluation are very much the same in that both emphasize obtaining student information concerning their conceptual understanding, their ability to use knowledge and reason to solve problems and their ability to communicate effectively.

RESOURCES

The challenge facing teachers today is implementing a mathematics program that will prepare students for a rapidly changing world. There is an increasing need for a variety of resources to support the teaching of the curriculum at all levels (entry-grade 12).

Students' learning resources and technologies used must support both the philosophy and recommended approaches of the mathematics curriculum and should be used in conjunction with a variety of other available teaching tools. These tools may include professional resources, kits, games, activity cards, manipulative materials, calculators and computer software. When purchasing commercial materials, it is important to be selective, choosing those that complement the curriculum focus and outcomes.

It should also be clear, given the need to implement new mathematical content, new instructional techniques and instructional materials such as those identified here, that on-going professional development will be needed.

PROFESSIONAL RESOURCES

Resources with which all teachers of mathematics should be familiar are publications of the National Council of Teachers of Mathematics (NCTM):

- *Curriculum And Evaluation Standards for School Mathematics* (1989)—represents NCTM's vision of what the mathematics curriculum should include in terms of content priority and emphasis
- *Professional Standards For Teaching Mathematics* (1991)—provides standards regarding what teachers must know in order to teach toward the new curriculum standards, recommendations for professional development and suggestions for the evaluation of the teaching of mathematics
- *Assessment Standards For School Mathematics* (1995)
- *Addenda to the Standards*
- *Yearbooks*

Professional publications help teachers by providing substantive ideas about mathematics teaching. Four recommended journals from NCTM are

- *Teaching Children Mathematics* (K-5)
- *Mathematics Teaching In The Middle School* (grades 6-8)
- *Mathematics Teacher* (grades 9-12)
- *Journal for Research in Mathematics Education*

MANIPULATIVE MATERIALS

The use of manipulative materials in the mathematics classroom supports the development of understanding in students of various ages and developmental levels. Manipulatives may be used to introduce new concepts, verify results and/or provide remedial help.

Every student should have the opportunity to explore with manipulative materials. Manipulatives should clearly represent the concept being taught; nevertheless, a particular manipulative may be sufficiently flexible for use in teaching a variety of concepts.

When possible, a variety of manipulatives should be used to develop each concept, the different representations helping students to generalize mathematical ideas. Students should as well express their understanding of the connection among the manipulatives, the mathematical concept and the mathematical symbols used to denote it.

TECHNOLOGY

"... New technology not only has made calculations and graphing easier, it has changed the very nature of the problems important to mathematics and the methods mathematicians use to investigate them."
(NCTM *Curriculum and Evaluation Standards*, p. 8)

Modern computing technologies have changed the way everyone learns and works. For many years, they have served as a catalyst for reform efforts in mathematics education. In today's elementary and secondary schools, they can and will continue to influence how mathematics is learned and taught, not only by making, for example, calculations and graphing easier and more manageable, but also by altering the very nature of what mathematics is important to learn. New problems as well as innovative ways of investigating them now become possible.

NCTM's *Curriculum and Evaluation Standards for School Mathematics* acknowledges the impact that technology has on mathematics and its many uses. The *Curriculum Standards* recommend the use of technology i) to enhance the teaching and the learning of mathematics and ii) to relate school mathematics to the world in which the students live through developing and interpreting mathematical models.

The introduction of computers, graphics calculators, video technology and other technologies into the mathematics classroom allows students to

- explore situations with complicated numbers which previously would have been beyond their capabilities
- explore individual or groups of related computations or functions quickly and easily
- create and explore numeric and geometric situations for the purpose of developing conjectures
- perform simulations of situations which would otherwise be impossible to examine
- easily link different representations of the same information
- model situations mathematically
- observe the effects of simple changes in parameters or coefficients
- analyze, organize and display data

All of these situations enhance discovery learning and problem-solving potential. At the same time, teachers have the opportunity to use technology to communicate with fellow mathematics teachers, to share lessons with experts and to expose their students to information that would otherwise be inaccessible.

As more sophisticated technology becomes accessible in classrooms, its productive use in support of the mathematics curriculum will need to be considered. In making such decisions, the opportunity for improved instruction and learning should be the guiding principle.

The effective use of technology in the mathematics classroom is not easy to achieve. Students will need to learn to make judgments as to when the use of technology is appropriate and when it is not. In all situations, it is imperative that it should be used both as a tool to include, rather than exclude, students and as a means of creating new teaching strategies.

References

Centre for Research in Applied Measurement and Evaluation. *Principles for Fair Student Assessment Practices for Education in Canada.* Edmonton, AB: University of Alberta, 1993.

National Council of Teachers of Mathematics. *Assessment Standards for School Mathematics.* Reston, VA: NCTM, 1995.

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: NCTM, 1989.

National Council of Teachers of Mathematics. *Estimation and Mental Computation: 1986 Yearbook.* Reston, VA: NCTM, 1986.

National Council of Teachers of Mathematics. *Professional Standards for Teaching Mathematics.* Reston, VA: NCTM, 1991.

Steen, et al. "Teaching Mathematics for Tomorrow's World." *Educational Leadership,* (September, 1989).

Van De Walle, John. *Elementary School Mathematics: Teaching Developmentally*. 2nd ed. New York: Longman, 1994.